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Does the quadratic equation have Greek roots? A study of "Geometric Algebra", "Application of Areas", and related problems. III.

Libertas Math. 2, 1-62 (1982).

[Chapters I and II have appeared ibid. 1, 1-49 (1981; Zbl. 475.01002). The following review concerns the Chapters I-IV.]

This strongly polemical article is a continuation of the first author's argument against writing the history of Ancient mathematics in the future perfect, in favour of a historicist method. It analyzes those parts of the Euclidean Elements (mainly books II and VI) which are often assumed to set forth a "geometric algebra", and argues that they cannot be understood as expositions of algebra in geometric disguise.

The argument is based on a post-Viètan concept of "algebra" which involves, inter alia, a fully abstract arithmetic as its underlying foundation. The operations underlying the "geometric algebra" are demonstrated not to correspond to such an arithmetical structure. So, multiplication by a number [πολλαπλασιασμός] and the formation of a rectangle are clearly different operations in the Elements; similarly, the application of an area and the consideration of a proportion are kept apart, although, arithmetically seen, both are divisions. So, the conceptual structure of the "geometric algebra" must be different from that of post-Viètan algebra. Similarly, by immanent analysis it is argued that the application of an area with excess or deficiency (Elements VI, 28 and 29) cannot be understood simply as aiming at the solution of a mixed second-degree equation.

The arguments that Greek "geometric arithmetic" and "geometric algebra" are not arithmetic and algebra in a modern abstract sense seem waterproof. However, the algebraic character of al-Khwârizmî's al-jabr and Leonardo Fibonacci's algebre could be discarded on the same grounds. So, the implications of the authors' investigation are restricted, and for a historicist reading of the words the term "geometric algebra" is not buried as definitively as stated. Especially, the possibility is not ruled out that the "geometric algebra" of Elements II and VI constitutes the endpoint of a theoretical development the beginning of which could have been inspired by Babylonian or post-Babylonian "algebra" - instead, the possibility is tacitly disregarded that Greek mathematics may have been transformed as a theoretical structure between the mid-fifth and the early third century B. C.

Concerning the relation to Babylonian "algebra", the authors are handicapped by relying exclusively on modernizing translations which conflate operations which are kept strictly apart in the original language. So, they come to regard Babylonian "algebra" as abstract arithmetic, which it hardly is, and they fail to notice that the Babylonian texts distinguish much the same classes of "multiplicative operations" as they find themselves when analyzing the methods of Greek "geometric algebra". J. Høyrup.